

STAT 2290 Homework 5

Hand in on April 8 or 9 with your quiz.

Read these instructions:

- Show your work. Answers without proper justifications receive no credit.

Sample proportions

Problem 1.

The Transportation Security Administration (TSA) is responsible for airport safety. On some flights, TSA officers randomly select passengers for an extra security check before boarding. One such flight had 76 passengers—12 in first class and 64 in coach class. TSA officers selected an SRS of 10 passengers for screening. Let \hat{p} be the proportion of first-class passengers in the sample.

- (a) Is the 10% condition met in this case? Justify your answer.
(b) Is the Large Counts condition met in this case? Justify your answer.

Problem 2.

Explain why you cannot use the methods of the lesson on "Sampling Distributions" to find the desired probability in the following situation:

A factory employs 3000 unionized workers, of whom 30% are Hispanic. The 15-member union executive committee contains 3 Hispanics. What would be the probability of 3 or fewer Hispanics if the executive committee were chosen at random from all the workers?

Problem 3.

Do you go to church? The Gallup Poll asked a random sample of 1785 adults whether they attended church during the past week. Let \hat{p} be the proportion of people in the sample who attended church. A newspaper report claims that 40% of all U.S. adults went to church last week. Suppose this claim is true.

- (a) What is the mean of the sampling distribution of \hat{p} ? Why?
(b) Find the standard deviation of the sampling distribution of \hat{p} . Check to see if the 10% condition is met.
(c) Is the sampling distribution of \hat{p} approximately normal? Check to see if the Large Counts condition is met.
(d) Of the poll respondents, 44% said they did attend church last week. Find the probability of obtaining a sample of 1785 adults in which 44% or more say they attended church last week if the newspaper report's claim is true. Does this poll give convincing evidence against the claim? Explain.
(e) What sample size would be required to reduce the standard deviation of the sampling distribution to one-third the value you found in Part (b)? Justify your answer.

Problem 4. (On-time shipping)

A mail-order company advertises that it ships 90% of its orders within three working days. You select an SRS of 100 of the 5000 orders received in the past week for an audit. The audit reveals that 86 of these orders were shipped on time.

- (a) If the company really ships 90% of its orders on time, what is the probability that the proportion in an SRS of 100 orders is 0.86 or less? Show your work.
(b) A critic says, "Aha! You claim 90%, but in your sample the on-time percentage is lower than that. So the 90% claim is wrong." Explain in simple language why your probability calculation in (a) shows that the result of the sample does not refute the 90% claim.

Sample mean (σ known)

Problem 5. (Dead battery?)

A car company has found that the lifetime of its batteries varies from car to car according to a Normal distribution with mean $\mu = 48$ months and standard deviation $\sigma = 8.2$ months. The company installs a new brand of battery on an SRS of 8 cars.

(a) If the new brand has the same lifetime distribution as the previous type of battery, describe the sampling distribution of the mean lifetime \bar{x} .

(b) The average life of the batteries on these 8 cars turns $\bar{x} = 42.2$ months. Find the probability that the sample mean lifetime is 42.2 months or less if the lifetime distribution is unchanged. What conclusion would you draw?

Problem 6. (Airline passengers get heavier)

In response to the increasing weight of airline passengers, the Federal Aviation Administration (FAA) told airlines to assume that passengers average 190 pounds in the summer, including clothes and carry-on baggage. But passengers vary, and the FAA did not specify a standard deviation. A reasonable standard deviation is 35 pounds. Weights are not Normally distributed, especially when the population includes both men and women, but they are not very non-Normal. A commuter plane carries 30 passengers.

(a) Explain why you cannot calculate the probability that a randomly selected passenger weighs more than 200 pounds from the above information alone.

(b) Find the probability that the total weight of 30 randomly selected passengers exceeds 6000 pounds. Show your work. (Hint: To apply the central limit theorem, restate the problem in terms of the mean weight.)